FREQUENCY AT WHICH VAPOR BUBBLES FORM DURING BOILING

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An expression is found for the frequency at which vapor bubbles form; the familiar empirical dependences follow from this expression as particular cases. The theoretical results are in satisfactory agreement with experiment.

There is no general solution for the frequency f at which vapor bubbles form during boiling [1, 2]. Experimental studies are reported in [3-10], and the efforts which have been made to determine f are reviewed in [1, 2, 11, 12].

In determing f, many investigators have sought its connection with the rupture diameter D in the following form:

$$fD = C$$

where the quantity C has frequentlybeen assumed a constant. According to [13], e.g., we have $C = 9.5 \cdot 10^{-2}$ m/sec; other values which have been reported are $7.87 \cdot 10^{2}$ m/sec for water and $7.62 \cdot 10^{-2}$ m/sec for CCl_4 [14], $8.38 \cdot 10^{-2}$ m/sec for methanol [15], 2.032 m/sec under natural-convection conditions during the boiling of a saturated liquid [16, 17], 0.1016 m/sec for methanol [18], and $8.8 \cdot 10^{-2}$ - $24.8 \cdot 10^{-2}$ m/sec during boiling of water at reduced pressure [10]. Contradictory dependences of C on the physical properties of the medium were reported in [12, 19-23].

It was shown in [24] that most of the published f and D results are not in satisfactory agreement with heat-transfer data because of incorrect averaging procedures: the arithmetic mean product $\langle fV \rangle$ increases with increasing heat flow q, and at certain q values we have fV = const for each bubble source.

Let us consider the growth of a bubble on the heating surface. During a time τ_d the bubble increases in size; after it reaches the rupture diameter D, it breaks off and enters the liquid. The volume it leaves is filled by a cooler liquid, which during a time τ_w is heated by an amount ΔT sufficient to begin the formation of a new bubble. The bubble formation frequency is

$$f = \frac{1}{\tau_w + \tau_d}$$

To determine the delay time τ_w we assume that the liquid coming into contact with the heating surface is a semiinfinite mass. In this case the solution of the heat-transfer equation in the liquid layer yields the following relationship [25] between the heat flow q, the temperature drop ΔT, and τ_w:

$$\tau_w{}^\circ = \frac{\tau_w}{\tau_*} = \frac{\pi}{4}\left(\frac{J}{P}\right)^2$$
$$J = \frac{\Delta T c' \rho'}{r \rho''}, \quad P = \frac{q R_*}{r \rho'' a'}, \quad \tau_* = \frac{R_*^2}{a'}, \quad R_* = \left(\frac{\sigma}{g(\rho' - \rho'')}\right)^{1/2} \qquad (1)$$

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To determine the bubble-growth time we use the equation

$$R = R_0 + \frac{4\varphi_\theta}{(2+f_\rho)\sqrt{\pi}}\, J\sqrt{a'\tau} + \frac{2}{2+f_\rho}\,\varphi_q\varphi_\rho\,\frac{q}{r\rho''}\,\tau \tag{2}$$

Then we find (for $R \gg R_0$)

$$\tau_d = \varphi_1^2\left(\frac{J}{P}\right)^2\left[\left(1+\varphi_2\frac{PR^\circ}{J^2}\right)^{1/2}-1\right]^2\tau_*, \quad R^\circ = \frac{R}{R_*}$$

$$\varphi_\theta = \frac{1+\cos\theta}{1+{}^1/_2\cos\theta\,(2+\sin^2\theta)}, \quad \varphi_q = \frac{1}{2}(1-\cos\theta), \quad f_\rho = 1-\frac{\rho''}{\rho'}$$

$$\varphi_1 = \frac{2}{\sqrt{\pi}\,(1-\cos\theta)}, \quad \varphi_2 = \frac{\pi(1-\cos\theta)\,(1+{}^1/_2 f_\rho)}{2\varphi_\theta}$$

For a large $J > 10$, Eq. (2) simplifies, and we can set

$$\tau_d = \frac{9\pi}{16}\,\frac{R^{\circ 2}}{J^2}\,\tau_* \tag{3}$$

The rupture radius R° is given by

$$R^{\circ 6} + \frac{3}{4}\,\frac{\zeta_w}{\zeta_g}\,N_w'\sin\alpha\,R^{\circ 5} + \left(\frac{3}{8}\,\frac{\zeta_w}{\zeta_g}\,\varphi_w N_w'\right)^2 R^{\circ 4} - \left(\frac{3}{2}\,\frac{\zeta_\sigma}{\zeta_g}\sin\theta\right)^2 R^{\circ 2} - \frac{8}{9\pi^2}$$
$$\times (1+\cos\theta)^2\sin\theta\,\frac{\zeta_R\zeta_\sigma}{\zeta_g^2}\,\vartheta J^4 R^\circ - \left[\frac{8(1+\cos\theta)}{27\pi^2}\,\frac{\zeta_R}{\zeta_g}\,\vartheta\right]^2 J^8 = 0$$

where

$$N_w' = 9\varepsilon\Phi N_w, \quad N_w = \frac{\rho' w^2}{\sqrt{\sigma g(\rho'-\rho'')}}$$

$$\vartheta = \frac{\rho' a'^2}{\sigma}\left(\frac{\sigma}{g(\rho'-\rho'')}\right)^{-1/2}, \quad \varepsilon = \frac{15}{14}(1+\cos\theta)^{1/2}$$

During boiling in a large volume on a horizontal surface, R° satisfies

$$R^{\circ 3} - \frac{3}{2}\,\frac{\zeta_\sigma\sin\theta}{\zeta_g}\,R^\circ - \frac{8(1+\cos\theta)^2\zeta_R\vartheta J^4}{27\pi^2\zeta_g} = 0 \tag{4}$$

Knowing R°, we can thus use Eqs. (1) and (3) to find the dimensionless rupture frequency $f^\circ = f\tau_*$; for large J, we have, approximately,

$$f^\circ = \left[\frac{\pi}{4}\left(\frac{J}{P}\right)^2 + \frac{9\pi}{16}\left(\frac{R^\circ}{J}\right)^2\right]^{-1} \tag{5}$$

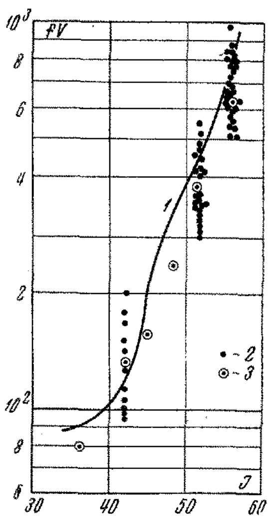

Fig. 1

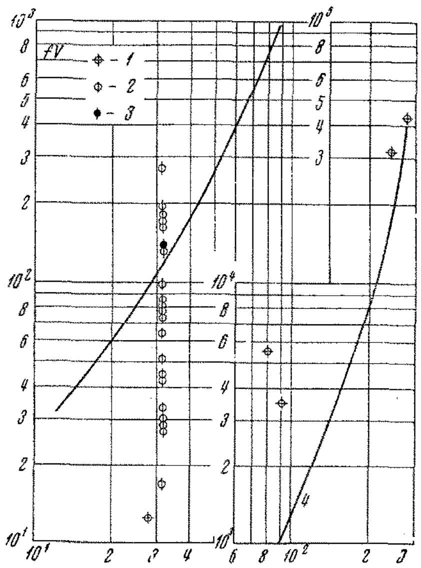

Fig. 2

or, more accurately,

$$f^0=\left(\frac{P}{J}\right)^2\left\{\frac{\pi}{4}+\varphi_1^2\left[\left(1+\varphi_2\frac{PR^\circ}{J^2}\right)^{1/2}-1\right]^2\right\}^{-1} \tag{6}$$

For J > 100, Eq. (4) yields $R^\circ \approx C_1 J^{4/3}$. In this case we can use Eq. (5) to find the equation analogous to the empirical dependence of [23], $fV \propto J^2$, for small q. If we have

$$^3/_2 PR^\circ/J^2 \gg 1 \tag{7}$$

we can find from Eqs. (5) and (6) an expression analogous to the empirical dependence of [22]:

$$f^\circ R^{\circ 2} = 4\pi^{-1}J^2$$

If $R^\circ \propto J^{4/3}$, we find as a particular case, the equation [20, 21]

$$fR^{1/2} = \mathrm{const}$$

If we have the inequality opposite (7), we find

$$f^\circ = \frac{4}{\pi}\left(\frac{P}{J}\right)^2$$

It follows that for small J, in which case R° is essentially independent of J, we have f = const, and at large J, for which we have $R^\circ \propto J^{4/3}$, we find $fR^{3/2}$ = const.

This brief analysis has shown that the exponent on R changes from 0 to 2; this result has been confirmed in particular cases [26].

An analysis [23, 24] has shown that we must seek the relationship between the frequency f and the volume of the detaching bubble, adopting as a parameter characterizing a particular vapor-formation center the product fV, the vapor-production rate of this center. To check these theoretical results, we therefore use plots of $f^\circ V^\bullet = f^\circ R^\circ$ vs J.

Figure 1 shows the experimental results of [24] (2), their average values (3), and a curve (1) calculated from the equations above. The agreement is seen to be satisfactory.

Figure 2 shows that the calculated results (3) and (4) agree with the experimental data of [10](1) and [27](2). (2).

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